24th Feb. 2021 | Shift - 2
MATHEMATICS

JEE | NEET | Foundation

MOTION™

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SELECTIONS SINCE 2007

1. Let $a, b \in R$. If the mirror image of the point $P(a, 6, 9)$ with respect to the line $\dfrac{x-3}{7} = \dfrac{y-2}{5} = \dfrac{z-1}{-9}$ is $(20, b, -a-9)$, then $|a+b|$ is equal to :

(1) 86　　　　　(2) 88　　　　　(3) 84　　　　　(4) 90

Ans. **(2)**

Sol. P(a, 6, 9), Q (20, b, −a−9)

mid point of $PQ = \left(\dfrac{a+20}{2}, \dfrac{b+6}{2}, -\dfrac{a}{2} \right)$

lie on line

$$\dfrac{\dfrac{a+20}{2} - 3}{7} = \dfrac{\dfrac{b+6}{2} - 2}{5} = \dfrac{-\dfrac{a}{2} - 1}{-9}$$

$$\dfrac{a+20-6}{14} = \dfrac{b+6-4}{10} = \dfrac{-a-2}{-18}$$

$$\dfrac{a+14}{14} = \dfrac{a+2}{18}$$

$18a + 252 = 14a + 28$

$4a = -224$

$\boxed{a = -56}$

$$\dfrac{b+2}{10} = \dfrac{a+2}{18}$$

$$\dfrac{b+2}{10} = \dfrac{-54}{18}$$

$$\dfrac{b+2}{10} = -3 \Rightarrow b = -32$$

$$|a+b| = |-56-32| = 88$$

2. Let f be a twice differentiable function defined on R such that $f(0) = 1, f'(0) = 2$ and $f'(x) \neq 0$ for all x∈R. If $\begin{vmatrix} f(x) & f'(x) \\ f'(x) & f''(x) \end{vmatrix} = 0$, for all x∈R then the value of $f(1)$ lies in the interval:

(1) (9, 12)　　　　　(2) (6, 9)　　　　　(3) (3, 6)　　　　　(4) (0, 3)

Ans. **(2)**

Sol. Given f(x) f ″(X) − (f′(x))^2 = 0

Let h (x) = $\dfrac{f(x)}{f'(x)}$

$\Rightarrow$ h'(x) = 0 $\qquad$ $\Rightarrow$ h(x) = k

$\Rightarrow \dfrac{f(x)}{f'(x)}$ = k $\qquad$ $\Rightarrow$ f (x) = k f'(x)

$\Rightarrow$ f(0) = k f'(0) $\qquad$ $\Rightarrow$ 1 = k(2) $\Rightarrow$ k = $\dfrac{1}{2}$

Now f(x) = $\dfrac{1}{2}$ f'(x) $\Rightarrow \int 2dx = \int \dfrac{f'(x)}{f(x)}dx$

$\Rightarrow$ 2x = ln|f(x)| + C
As f(0) = 1 $\Rightarrow$ C = 0
$\Rightarrow$ 2x = ln|f(X)| $\Rightarrow$ f(x) = $\pm$ e^{2x}
As f(0) = 1 $\Rightarrow$ f(x) = e^{2x} $\Rightarrow$ f(1) = e^2

3. A possible value of $\tan\left(\dfrac{1}{4}\sin^{-1}\dfrac{\sqrt{63}}{8}\right)$ is:

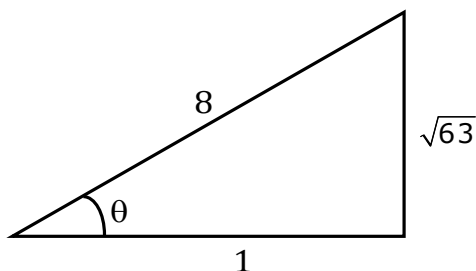(1) $\dfrac{1}{2\sqrt{2}}$ $\qquad$ (2) $\dfrac{1}{\sqrt{7}}$ $\qquad$ (3) $\sqrt{7}-1$ $\qquad$ (4) $2\sqrt{2}-1$

Ans. **(2)**

Sol. $\tan\left(\dfrac{1}{4}\sin^{-1}\dfrac{\sqrt{63}}{8}\right)$

Let $\sin^{-1}\left(\dfrac{\sqrt{63}}{8}\right)=\theta$ $\qquad$ $\sin\theta = \dfrac{\sqrt{63}}{8}$

$\cos\theta = \dfrac{1}{8}$

$2\cos^2\dfrac{\theta}{2}-1 = \dfrac{1}{8}$

$$\cos^2 \frac{\theta}{2} = \frac{9}{16}$$

$$\cos \frac{\theta}{2} = \frac{3}{4}$$

$$\frac{1 - \tan^2 \frac{\theta}{4}}{1 + \tan^2 \frac{\theta}{4}} = \frac{3}{4}$$

$$\tan \frac{\theta}{4} = \frac{1}{\sqrt{7}}$$

4. The probability that two randomly selected subsets of the set {1,2,3,4,5} have exactly two elements in their intersection, is:

(1) $\dfrac{65}{2^7}$ (2) $\dfrac{135}{2^9}$ (3) $\dfrac{65}{2^8}$ (4) $\dfrac{35}{2^7}$

Ans. (2)

Sol. Required probability

$$= \frac{^5C_2 \times 3^3}{4^5}$$

$$= \frac{10 \times 27}{2^{10}} = \frac{135}{2^9}$$

5. The vector equation of the plane passing through the intersection of the planes $\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 1$ and $\vec{r} \cdot (\hat{i} - 2\hat{j}) = -2,$ and the point (1,0,2) is :

(1) $\vec{r} \cdot (\hat{i} - 7\hat{j} + 3\hat{k}) = \dfrac{7}{3}$ (2) $\vec{r} \cdot (\hat{i} + 7\hat{j} + 3\hat{k}) = 7$

(3) $\vec{r} \cdot (3\hat{i} + 7\hat{j} + 3\hat{k}) = 7$ (4) $\vec{r} \cdot (\hat{i} + 7\hat{j} + 3\hat{k}) = \dfrac{7}{3}$

Ans. (2)

Sol. Plane passing through intersection of plane is

$$\left\{ \vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) - 1 \right\} + \lambda \left\{ \vec{r} \cdot (\hat{i} - 2\hat{j}) + 2 \right\} = 0$$

Passes through $\hat{i} + 2\hat{k}$, we get

$(3-1) + \lambda(1+2) = 0 \Rightarrow \lambda = -\dfrac{2}{3}$

Hence, equation of plane is $3\left\{\vec{r} \cdot \left(\hat{i} + \hat{j} + \hat{k}\right) - 1\right\} - 2\left\{\vec{r} \cdot \left(\hat{i} - 2\hat{j}\right) + 2\right\} = 0$

$\Rightarrow \vec{r} \cdot \left(\hat{i} + 7\hat{j} + 3\hat{k}\right) = 7$
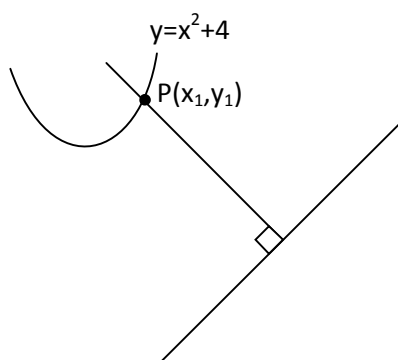
6. If P is a point on the parabola $y = x^2 + 4$ which is closest to the straight line $y = 4x - 1$, then the co-ordinates of P are :

(1) (−2, 8)　　　　(2) (1, 5)　　　　(3) (3, 13)　　　　(4) (2, 8)

Ans. **(4)**

Sol.　$\dfrac{dy}{dx}\Big|_p = 4$

∴ $2x_1 = 4$

$\Rightarrow x_1 = 2$

∴ Point will be (2, 8)

7. Let a, b, c be in arithmetic progression. Let the centroid of the triangle with vertices $(a, c), (2, b)$ and (a, b) be $\left(\dfrac{10}{3}, \dfrac{7}{3}\right)$. If α, β are the roots of the equation $ax^2 + bx + 1 = 0$, then the value of $\alpha^2 + \beta^2 - \alpha\beta$ is:

(1) $\dfrac{71}{256}$　　　　(2) $-\dfrac{69}{256}$　　　　(3) $\dfrac{69}{256}$　　　　(4) $-\dfrac{71}{256}$

Ans. **(4)**

Sol.　$2b = a + c$

$\dfrac{2a + 2}{3} = \dfrac{10}{3}$ and $\dfrac{2b + c}{3} = \dfrac{7}{3}$

$a = 4$, $\left.\begin{matrix} 2b + c = 7 \\ 2b - c = 4 \end{matrix}\right\}$, solving

$b = \dfrac{11}{4}$

$c = \dfrac{3}{2}$

∴ Quadratic Equation is $4x^2 + \dfrac{11}{4}x + 1 = 0$

∴ The value of $(\alpha + \beta)^2 - 3\alpha\beta = \dfrac{121}{256} - \dfrac{3}{4} = -\dfrac{71}{256}$

8. The value of the integral, $\displaystyle\int_1^3 \left[x^2 - 2x - 2\right]dx$, where [x] denotes the greatest integer less than or equal to x, is:

(1) −4   (2) −5   (3) $-\sqrt{2} - \sqrt{3} - 1$ (4) $-\sqrt{2} - \sqrt{3} + 1$

Ans. (3)

Sol. $I = \displaystyle\int_1^3 -3\,dx + \int_1^3 \left[(x-1)^2\right]dx$

Put $x - 1 = t$; dx = dt

$I = (-6) + \displaystyle\int_0^2 \left[t^2\right]dt$

$I = -6 + \displaystyle\int_0^1 0\,dt + \int_1^{\sqrt{2}} 1\,dt + \int_{\sqrt{2}}^{\sqrt{3}} 2\,dt + \int_{\sqrt{3}}^2 3\,dt$

$I = -6 + \left(\sqrt{2} - 1\right) + 2\sqrt{3} - 2\sqrt{2} + 6 - 3\sqrt{3}$

$I = -1 - \sqrt{2} - \sqrt{3}$

9. Let $f : \mathbf{R} \to \mathbf{R}$ be defined as

$$f(x) = \begin{cases} -55x, & \text{if } x < -5 \\ 2x^3 - 3x^2 - 120x, & \text{if } -5 \le x \le 4 \\ 2x^3 - 3x^2 - 36x - 336, & \text{if } x > 4 \end{cases}$$

Let $A = \{x \in R : f \text{ is increasing}\}$. Then A is equal to :

(1) $(-5, -4) \cup (4, \infty)$ (2) $(-5, \infty)$

(3) $(-\infty, -5) \cup (4, \infty)$ (4) $(-\infty, -5) \cup (-4, \infty)$

Ans. (1)

Sol. $f(x) = \begin{cases} -55 & ; & x < -5 \\ 6(x^2 - x - 20) & ; & -5 < x < 4 \\ 6(x^2 - x - 6) & ; & x > 4 \end{cases}$

$f(x) = \begin{cases} -55 & ; & x < -5 \\ 6(x-5)(x+4) & ; & -5 < x < 4 \\ 6(x-3)(x+2) & ; & x > 4 \end{cases}$

Hence, f(x) is monotonically increasing in interval $(-5, -4) \cup (4, \infty)$

10. If the curve $y = ax^2 + bx + c, x \in R,$ passes through the point (1,2) and the tangent line to this curve at origin is $y = x$, then the possible values of a,b,c are :

(1) a =1, b=1, c=0   (2) a= −1, b=1, c =1

(3) a =1, b=0, c =1   (4) $a = \dfrac{1}{2}, b = \dfrac{1}{2}, c = 1$

Ans. (1)

Sol. 2 = a + b + c ……(i)

$\dfrac{dy}{dx} = 2ax + b \Rightarrow \dfrac{dy}{dx}\Big|_{(0,0)} = 1$

$\Rightarrow b = 1 \Rightarrow a + c = 1$

(0,0) lie on curve

$\therefore c=0, a=1$

11. The negation of the statement

$\sim p \wedge (p \vee q)$ is :

(1) $\sim p \wedge q$ (2) $p \wedge \sim q$ (3) $\sim p \vee q$ (4) $p \vee \sim q$

Ans. (4)

Sol.

p	q	~ p	p ∨ q	(~ p) ∧ (p ∨ q)	~ q	p ∨ ~ q
T	T	F	T	F	F	T
T	F	F	T	F	T	T
F	T	T	T	T	F	F
F	F	T	F	F	T	T

∴ ~ $p \wedge (p \vee q) \equiv p \vee \sim q$

12. For the system of linear equations:

$x - 2y = 1, x - y + kz = -2, ky + 4z = 6, k \in \mathbf{R}$

consider the following statements:

(A) The system has unique solution if $k \neq 2, k \neq -2$.

(B) The system has unique solution if $k = -2$.

(C) The system has unique solution if $k = 2$.

(D) The system has no-solution if $k = 2$.

(E) The system has infinite number of solutions if $k \neq -2$.

Which of the following statements are correct?

(1) (B) and (E) only   (2) (C) and (D) only

(3) (A) and (D) only   (4) (A) and (E) only

Ans. **(3)**

Sol. $x - 2y + 0.z = 1$

$x - y + kz = -2$

$0.x + ky + 4z = 6$

$$\Delta = \begin{vmatrix} 1 & -2 & 0 \\ 1 & -1 & k \\ 0 & k & 4 \end{vmatrix} = 4 - k^2$$

For unique solution $4 - k^2 \neq 0$

$$\boxed{k \neq \pm 2}$$

For k=2

$x - 2y + 0.z = 1$

$x - y + 2z = -2$

$0.x + 2y + 4z = 6$

$$\Delta x = \begin{vmatrix} 1 & -2 & 0 \\ -2 & -1 & 2 \\ 6 & 2 & 4 \end{vmatrix} = (-8) + 2[-20]$$

$\Delta x = -48 \neq 0$

For k=2 $\Delta x \neq 0$

For K=2; The system has no solution

13. For which of the following curves, the line $x + \sqrt{3}y = 2\sqrt{3}$ is the tangent at the point $\left(\dfrac{3\sqrt{3}}{2}, \dfrac{1}{2}\right)$?

(1) $x^2 + 9y^2 = 9$ (2) $2x^2 - 18y^2 = 9$

(3) $y^2 = \dfrac{1}{6\sqrt{3}} x$ (4) $x^2 + y^2 = 7$

Ans. (1)

Sol. Tangent to $x^2 + 9y^2 = 9$ at point $\left(\dfrac{3\sqrt{3}}{2}, \dfrac{1}{2}\right)$ is $x \left|\dfrac{3\sqrt{3}}{2}\right| + 9y\left(\dfrac{1}{2}\right) = 9$

$3\sqrt{3}x + 9y = 18 \Rightarrow x + \sqrt{3}y = 2\sqrt{3}$

$\Rightarrow$ option (1) is true

14. The angle of elevation of a jet plane from a point A on the ground is 60°. After a flight of 20 seconds at the speed of 432 km/ hour, the angle of elevation changes to 30°. If the jet plane is flying at a constant height, then its height is:
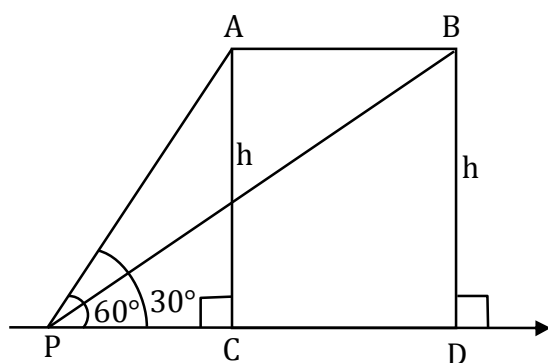
(1) $1200\sqrt{3}$m (2) $1800\sqrt{3}$m (3) $3600\sqrt{3}$m (4) $2400\sqrt{3}$m

Ans. (1)

Sol.

$v = 432 \times \dfrac{1000}{60 \times 60}$ m/sec = 120 m/sec

Distance AB = v × 20 = 2400 meter

In $\triangle$PAC

$\tan 60° = \dfrac{h}{PC} \Rightarrow PC = \dfrac{h}{\sqrt{3}}$

In $\triangle$PBD

$\tan 30° = \dfrac{h}{PD} \Rightarrow PD = \sqrt{3}h$

PD = PC + CD

$\sqrt{3}h = \dfrac{h}{\sqrt{3}} + 2400 \Rightarrow \dfrac{2h}{\sqrt{3}} = 2400$

h = 1200 $\sqrt{3}$ meter

15. For the statements p and q, consider the following compound statements:

(a) $(\sim q \wedge (p \to q)) \to \sim p$

(b) $((p \vee q) \wedge \sim p) \to q$

Then which of the following statements is correct?

(1) (a) is a tautology but not (b)

(2) (a) and (b) both are not tautologies.

(3) (a) and (b) both are tautologies.

(4) (b) is a tautology but not (a).

Ans. (3)

Sol. (a)

p	q	$\sim q$	$p \to q$	$\sim q \wedge (p \to q)$	$\sim p$	$(\sim q) \wedge (p \to q) \to \sim p$
T	T	F	T	F	F	T
T	F	T	F	F	F	T
F	T	F	T	F	T	T
F	F	T	T	T	T	T

(a) is tautologies

(b)

p	q	$p \vee q$	$\sim p$	$(p \vee q) \wedge \sim p$	$((p \vee q) \wedge \sim p) \to q$
T	T	T	F	F	T
T	F	T	F	F	T
F	T	T	T	T	T
F	F	F	T	F	T

(b) is tautologies

∴ a & b are both tautologies.

16. Let A and B be 3×3 real matrices such that A is symmetric matrix and B is skew-symmetric matrix. Then the system of linear equations $\left(A^2 B^2 - B^2 A^2\right) X = O$, where X is a 3×1 column matrix of unknown variables and O is a 3×1 null matrix, has :

(1) a unique solution   (2) exactly two solutions

(3) infinitely many solutions   (4) no solution

Ans. (3)

Sol. $A^T = A, B^T = -B$

Let $A^2 B^2 - B^2 A^2 = P$

$P^T = (A^2B^2 - B^2A^2)^T = (A^2B^2)^T - (B^2A^2)^T$

$= (B^2)^T (A^2)^T - (A^2)^T (B^2)^T$

$= B^2A^2 - A^2B^2$

$\Rightarrow$ P is skew-symmetric matrix

$$\begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

∴ $ay + bz = 0$ …(1)

         $-ax + cz = 0$ …(2)

         $-bx - cy = 0$ …(3)

From equation 1,2,3

$\Delta = 0$ & $\Delta_1 = \Delta_2 = \Delta_3 = 0$

∴ equation have infinite number of solution

17. If $n \geq 2$ is a positive integer, then the sum of the series

$^{n+1}C_2 + 2\left(^2C_2 + {}^3C_2 + {}^4C_2 + \ldots + {}^nC_2\right)$ is :

(1) $\dfrac{n(n+1)^2(n+2)}{12}$ (2) $\dfrac{n(n-1)(2n+1)}{6}$

(3) $\dfrac{n(n+1)(2n+1)}{6}$ (4) $\dfrac{n(2n+1)(3n+1)}{6}$

Ans. (3)

Sol. $^2C_2 = {}^3C_3$

$S = {}^3C_3 + {}^3C_2 + \ldots\ldots + {}^nC_2 = {}^{n+1}C_3$

$\because {}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

$\therefore {}^{n+1}C_2 + {}^{n+1}C_3 + {}^{n+1}C_3 = {}^{n+2}C_3 + {}^{n+1}C_3$

$= \dfrac{(n+1)!}{3!(n-1)!} + \dfrac{(n+1)!}{3!(n-2)!}$

$= \dfrac{(n+2)(n+1)n}{6} + \dfrac{(n+1)(n)(n-1)}{6} = \dfrac{n(n+1)(2n+1)}{6}$

18. If a curve $y = f(x)$ passes through the point (1,2) and satisfies $x\dfrac{dy}{dx} + y = bx^4$, then for what value of b, $\displaystyle\int_1^2 f(x)dx = \dfrac{62}{5}$?

(1) 5   (2) $\dfrac{62}{5}$ (3) $\dfrac{31}{5}$ (4) 10

Ans. **(4)**

Sol. $\dfrac{dy}{dx} + \dfrac{y}{x} = bx^3$, I.F. $= e^{\int \frac{dx}{x}} = x$

$\therefore yx = \int bx^4 dx = \dfrac{bx^5}{5} + C$

Passes through (1,2), we get

$2 = \dfrac{b}{5} + C$ …..(i)

Also, $\displaystyle\int_1^2 \left(\dfrac{bx^4}{5} + \dfrac{c}{x} \right) dx = \dfrac{62}{5}$

$\Rightarrow \dfrac{b}{25} \times 32 + C\ln 2 - \dfrac{b}{25} = \dfrac{62}{5} \Rightarrow C = 0 \ \& \ b = 10$

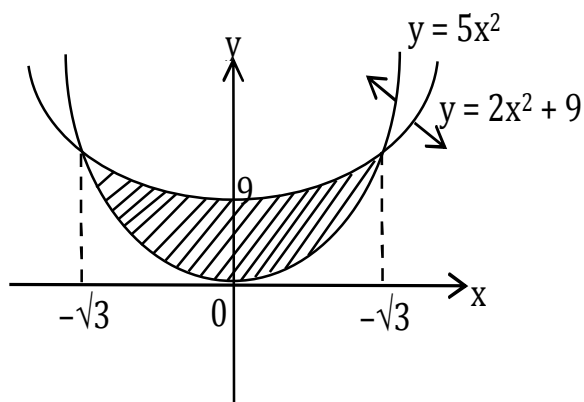**19.** The area of the region : $R = \{(x, y) : 5x^2 \le y \le 2x^2 + 9\}$ is:

(1) $9\sqrt{3}$ square units   (2) $12\sqrt{3}$ square units (3) $11\sqrt{3}$ square units (4) $6\sqrt{3}$ square units

Ans. **(2)**

Sol.

Required area

$$= 2\int_{0}^{\sqrt{3}} \left(2x^2 + 9 - 5x^2\right)dx$$

$$= 2\int_{0}^{\sqrt{3}} \left(9 - 3x^2\right)dx$$

$$= 2\,|\,9x - x^3\,|_{0}^{\sqrt{3}} = 12\sqrt{3}$$

20. Let $f(x)$ be a differentiable function defined on $[0,2]$ such that $f'(x) = f'(2-x)$ for all $x \in (0,2), f(0) = 1$ and $f(2) = e^2$. Then the value of $\int_{0}^{2} f(x)dx$ is:

(1) $1 + e^2$ (2) $1 - e^2$ (3) $2(1- e^2)$ (4) $2(1+e^2)$

Ans. **(1)**

Sol.   f '(x) = f '(2−x)

On integrating both side f(x) = −f(2−x) +c

put x = 0

f(0) + f(2) = c $\Rightarrow$ c = 1 + e^2

$\Rightarrow$ f(x) + f(2−x) = 1+e^2 ......(i)

$$I = \int_{0}^{2} f(x)dx = \int_{0}^{1}\left\{f(x) + f(2-x)\right\}dx = \left(1+e^2\right)$$

Section B

1.   The number of the real roots of the equation $(x+1)^2+|x-5|=\dfrac{27}{4}$ is_____.

Ans.   2

Sol. $x \geq 5$

$$(x+1)^2+(x-5)=\frac{27}{4}$$

$$\Rightarrow x^2+3x-4=\frac{27}{4}$$

$$\Rightarrow x^2+3x-\frac{43}{4}=0$$

$$\Rightarrow 4x^2+12x-43=0$$

$$x=\frac{-12\pm\sqrt{144+688}}{8}$$

$$x=\frac{-12\pm\sqrt{832}}{8}=\frac{-12\pm28.8}{8}$$

$$=\frac{-3\pm7.2}{2}$$

$$=\frac{-3+7.2}{2},\frac{-3-7.2}{2} \quad \text{(Therefore no solution)}$$

For $x \leq 5$

$$(x+1)^2-(x-5)=\frac{27}{4}$$

$$x^2+x+6-\frac{27}{4}=0$$

$$4x^2+4x-3=0$$

$$x=\frac{-4\pm\sqrt{16+48}}{8}$$

$$x=\frac{-4\pm8}{8}\Rightarrow x=-\frac{12}{8},\frac{4}{8}$$

$\therefore$ 2 Real Root's

2. The students $S_1, S_2, \ldots, S_{10}$ are to be divided into 3 groups A, B and C such that each group has at least one student and the group C has at most 3 students. Then the total number of possibilities of forming such groups is_____.

Ans. **31650**

Sol.

C→1   9 $\left[\begin{array}{l} A \\ B \end{array}\right.$

C→2   8 $\left[\begin{array}{l} A \\ B \end{array}\right.$

C→3   7 $\left[\begin{array}{l} A \\ B \end{array}\right.$

$= {}^{10}C_1 [2^9 - 2] + {}^{10}C_2[2^8 - 2] + {}^{10}C_3 [2^7 - 2]$

$= 2^7[{}^{10}C_1 \times 4 + {}^{10}C_2 \times 2 + {}^{10}C_3] - 20 - 90 - 240$

$= 128 [40 + 90 + 120] - 350$

$= (128 \times 250) - 350$

$= 10[3165] = 31650$

3. If $a + \alpha = 1, b + \beta = 2$ and $af(x) + \alpha f\left(\dfrac{1}{x}\right) = bx + \dfrac{\beta}{x}, x \neq 0,$ then the value of the expression

$\dfrac{f(x) + f\left(\dfrac{1}{x}\right)}{x + \dfrac{1}{x}}$ is_____.

Ans. **2**

Sol. $af(x) + \alpha f\left(\dfrac{1}{x}\right) = bx + \dfrac{\beta}{x}$ ……………(i)

$x \to \dfrac{1}{x}$

$af\left(\dfrac{1}{x}\right) + \alpha f(x) = \dfrac{b}{x} + \beta x$ ……………(ii)

(i) + (ii)

$(a + \alpha)\left[f(x) + f\left(\dfrac{1}{x}\right)\right] = \left(x + \dfrac{1}{x}\right)(b + \beta)$

$$\frac{f(x) + f\left(\frac{1}{x}\right)}{x + \frac{1}{x}} = \frac{2}{1} = 2$$

4. If the variance of 10 natural numbers $1,1,1,\ldots,1,k$ is less than 10, then the maximum possible value of k is_____.

Ans. **11**

Sol. $\sigma^2 = \dfrac{\Sigma x^2}{n} - \left(\dfrac{\Sigma x}{n}\right)^2$

$\sigma^2 = \dfrac{\left(9 + k^2\right)}{10} - \left(\dfrac{9 + k}{10}\right)^2 < 10$

$(90 + k^2)\, 10 - (81 + k^2 + 8k) < 1000$

$90 + 10k^2 - k^2 - 18k - 81 < 1000$

$9k^2 - 18k + 9 < 1000$

$(k - 1)^2 < \dfrac{1000}{9} \Rightarrow k - 1 < \dfrac{10\sqrt{10}}{3}$

$k < \dfrac{10\sqrt{10}}{3} + 1$

Maximum integral value of k = 11

5. Let λ be an integer. If the shortest distance between the lines $x - \lambda = 2y - 1 = -2z$ and $x = y + 2\lambda = z - \lambda$ is $\dfrac{\sqrt{7}}{2\sqrt{2}}$, then the value of $|\lambda|$ is

Ans. **1**

Sol. $\dfrac{x - \lambda}{1} = \dfrac{y - \dfrac{1}{2}}{\dfrac{1}{2}} = \dfrac{z}{-\dfrac{1}{2}}$

$\dfrac{x - \lambda}{2} = \dfrac{y - \dfrac{1}{2}}{1} = \dfrac{2}{-1}$　.....(1)　　Point on line $= \left(\lambda, \dfrac{1}{2}, 0\right)$

$\dfrac{x}{1} = \dfrac{y + 2\lambda}{1} = \dfrac{z - \lambda}{1}$　.....(2)　　Point on line $= \left(0, -2\lambda, \lambda\right)$

Distance between skew lines = $\dfrac{\left[\vec{a}_2 - \vec{a}_1 \ \ \vec{b}_1 \ \ \vec{b}_2\right]}{\left|\vec{b}_1 \times \vec{b}_2\right|}$

$$\begin{vmatrix} \lambda & \dfrac{1}{2}+2\lambda & -\lambda \\ 2 & 1 & -1 \\ 1 & 1 & 1 \end{vmatrix}$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 1 & 1 & 1 \end{vmatrix}$$

$= \dfrac{\left|-5\lambda - \dfrac{3}{2}\right|}{\sqrt{14}} = \dfrac{\sqrt{7}}{2\sqrt{2}}$ (given)

$= \left|10\lambda + 3\right| = 7 \Rightarrow \lambda = -1$

$\Rightarrow \left|\lambda\right| = 1$

6. Let $i = \sqrt{-1}$. If $\dfrac{(-1+i\sqrt{3})^{21}}{(1-i)^{24}} + \dfrac{(1+i\sqrt{3})^{21}}{(1+i)^{24}} = k$, and $n = \left[\left|k\right|\right]$ be the greatest integral part of $|k|$.

Then $\displaystyle\sum_{j=0}^{n+5}(j+5)^2 - \sum_{j=0}^{n+5}(j+5)$ is equal to_____.

Ans. 310

Sol. $\dfrac{\left(2e^{i\frac{2\pi}{3}}\right)^{21}}{\left(\sqrt{2}e^{-i\frac{\pi}{4}}\right)^{24}} + \dfrac{\left(2e^{i\frac{\pi}{3}}\right)^{21}}{\left(\sqrt{2}e^{i\frac{\pi}{4}}\right)^{24}}$

$\Rightarrow \dfrac{2^{21}.e^{i14\pi}}{2^{12}.e^{-i6\pi}} + \dfrac{2^{21}\left(e^{i7\pi}\right)}{2^{12}\left(e^{i6\pi}\right)}$

$\Rightarrow 2^9 e^{i(20\pi)} + 2^9 e^{i\pi}$

$\Rightarrow 2^9 + 2^9(-1) = 0$

n= 0

$$\sum_{j=0}^{5}(j+5)^2 - \sum_{j=0}^{5}(j+5)$$

$$\Rightarrow \left[5^2+6^2+7^2+8^2+9^2+10^2\right]-\left[5+6+7+8+9+10\right]$$

$$\Rightarrow \left[\left(1^2+2^2+....+10^2\right)-\left(1^2+2^2+3^2+4^2\right)\right]-\left[\left(1+2+3+.....+10\right)-\left(1+2+3+4\right)\right]$$

$$\Rightarrow (385-30)-[55-10]$$

$$\Rightarrow 355-45 \Rightarrow 310 \text{ ans.}$$

7. Let a point P be such that its distance from the point (5,0) is thrice the distance of P from the point $(-5,0)$. If the locus of the point P is a circle of radius r, then $4r^2$ is equal to

Ans. **56**

Sol. Let P(h,k)

Given

PA = 3PB

$PA^2 = 9PB^2$

$\Rightarrow (h-5)^2 + k^2 = 9[(h+5)^2+k^2]$

$\Rightarrow 8h^2 + 8k^2 + 100h + 200 = 0$

∴ Locus

$x^2 + y^2 + \left(\dfrac{25}{2}\right)x + 25 = 0$

∴ $c \equiv \left(\dfrac{-25}{4}, 0\right)$

∴ $r^2 = \left(\dfrac{-25}{4}\right)^2 - 25$

$= \dfrac{625}{16} - 25$

$= \dfrac{225}{16}$

∴ $4r^2 = 4 \times \dfrac{225}{16} = \dfrac{225}{4} = 56.25$

After Round of $4r^2 = 56$

8. For integers n and r, let $\begin{pmatrix} n \\ r \end{pmatrix} = \begin{cases} ^nC_r, & \text{if } n \geq r \geq 0 \\ 0, & \text{otherwise} \end{cases}$

The maximum value of k for which the sum

$\sum_{i=0}^{k}\begin{pmatrix} 10 \\ i \end{pmatrix}\begin{pmatrix} 15 \\ k-i \end{pmatrix} + \sum_{i=0}^{k+1}\begin{pmatrix} 12 \\ i \end{pmatrix}\begin{pmatrix} 13 \\ k+1-i \end{pmatrix}$ exists, is equal to_____.

Ans. **Bonus**

Sol. $(1+x)^{10} = {}^{10}C_0 + {}^{10}C_1x + {}^{10}C_2x^2 + \ldots\ldots + {}^{10}C_{10}x^{10}$

$(1+x)^{15} = {}^{15}C_0 + {}^{15}C_1x + \ldots{}^{15}C_{k-1}x^{k-1} + {}^{15}C_kx^k + {}^{15}C_{k+1}x^{k+1}+\ldots\ldots {}^{15}C_{15}x^{15}$

$\sum_{i=0}^{k}(10C_i)(15C_{k-i}) = {}^{10}C_0 \cdot {}^{15}C_k + {}^{10}C_1 \cdot {}^{15}C_{k-1}+\ldots..+{}^{10}C_k \cdot {}^{15}C_0$

Coefficient of x_k in $(1+x)^{25}$

$= {}^{25}C_k$

$\sum_{i=0}^{k+1}({}^{12}C_i)({}^{13}C_{k+1-i}) = {}^{12}C_0 \cdot {}^{13}C_{k+1} + {}^{12}C_1 \cdot {}^{13}C_k +\ldots\ldots+ {}^{12}C_{k+1} \cdot {}^{13}C_0$

Coefficient of x^{k+1} in $(1+x)^{25}$

$= {}^{25}C_{k+1}$

$^{25}C_k + {}^{25}C_{k+1} = {}^{26}C_{k+1}$

As ^nC_r is defined for all values of n as will as r. so $^{26}C_{k+1}$ always exist

Now k is unbounded so maximum values is not defined.

9. The sum of first four terms of a geometric progression (G.P.) is $\frac{65}{12}$ and the sum of their

respective reciprocals is $\frac{65}{18}$. If the product of first three terms of the G.P. is 1, and the third

term is α, then 2α is_____.

Ans. **3**

Sol. a, ar, ar^2, ar^3

$a + ar + ar^2 + ar^3 = \frac{65}{12}$ ………….(1)

$\frac{1}{a} + \frac{1}{ar} + \frac{1}{ar^2} + \frac{1}{ar^3} = \frac{65}{18}$

$\frac{1}{a}\left(\frac{r^3 + r^2 + r + 1}{r^3}\right) = \frac{65}{18}$ ………….(2)

$\dfrac{(i)}{(ii)}, a^2 r^3 = \dfrac{18}{12} = \dfrac{3}{2}$

$a^3 r^3 = 1 \Rightarrow a\left(\dfrac{3}{2}\right) = 1 \Rightarrow a = \dfrac{2}{3}$

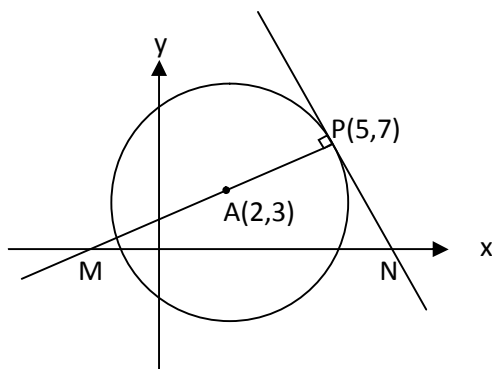$\dfrac{4}{9} r^3 = \dfrac{3}{2} \Rightarrow r^3 = \dfrac{3^3}{2^3} \Rightarrow r = \dfrac{3}{2}$

$\alpha = ar^2 = \dfrac{2}{3} \cdot \left(\dfrac{3}{2}\right)^2 = \dfrac{3}{2}$

$2\alpha = 3$

10. If the area of the triangle formed by the positive x-axis, the normal and the tangent to the circle $(x-2)^2 + (y-3)^2 = 25$ at the point (5,7) is A, then 24A is equal to_____.

Ans. **Bonus**

Sol.

Equation of normal at P

$(y-7) = \left(\dfrac{7-3}{5-2}\right)(x-5)$

3y −21 = 4x − 20

$\Rightarrow$ 4x − 3y + 1 = 0 ....(i)

$\Rightarrow$ $M\left(-\dfrac{1}{4}, 0\right)$

Equation of tangent at P

$(y-7) = -\dfrac{3}{4}(x-5)$

4y −28 = −3x + 15

$\Rightarrow$ 3x + 4y = 43 ....(ii)

$\Rightarrow$ $N\left(\dfrac{43}{3}, 0\right)$

Hence ar $(\Delta PMN) = \dfrac{1}{2} \times MN \times 7$

$A = \dfrac{1}{2} \times \dfrac{175}{12} \times 7$

$\Rightarrow 24A = 1225$

As positive x- axis is given in the question so question should be bonus.